

On the Quantum Theoretic Implications of Newton's Alchemy

by Alex Kasman

Only the skin around Rick's *left* eye wrinkled when he gave me the good news. That's a bad sign. Normally, the crow's feet around both eyes become more pronounced when he smiles. They don't make him look old, just extremely happy. But occasionally the wrinkles are only visible on one eye or the other.

I must have witnessed this twenty times during the four years that Rick served as my thesis advisor. I had also observed a few examples of these one-sided smiles during the previous year, when he taught my *Topics in Mathematical Analysis* class. Based on this empirical evidence, I developed a theory to explain this "broken symmetry". I was pretty sure that his non-symmetric, smile-induced wrinkles are indicative of a *forced* smile, one that accompanies a lie.

For instance, if one of the two bright students in his statistics class made a particularly insightful remark during class, I think Rick would have liked to have said "That's fantastic! You're a talented young mathematician. Have you considered majoring in math?" But, worried that this would bloat their egos or offend the other students, he would instead say instead "Yes, that's correct". This sort of remark was generally accompanied by wrinkles around his right eye.

On the other hand, sometimes he had to smile when he didn't feel like it. Comments like "The department has voted to let me serve as chair again starting next semester" or "Despite your low grades so far, if you study hard before the final you can still earn a good grade in this course" are the sorts of things he is likely to be saying while smiling with wrinkles around his left-eye only.

So, I was a bit concerned when he told me of the job offer from Dr. Stein. On the surface, it was good news. Despite the difficult job market, especially for people in my field of non-standard analysis, I had at least one firm job offer.

But only the skin around Rick's left eye wrinkled.

"What do you think?" he asked me as we walked through the flurry of cherry blossom petals on the central quad. "Does that fit in with your plans?"

As if I *had* plans! My plan was to get my degree and get a job. If this was a job, then it fit with my plans. Anyway, I received no other job offers, and so had no choices to consider. Not only that, but it was a research postdoc at a mathematics institute! And, from what I had heard, Ann Arbor isn't such a bad place to be.

As I said, it seemed to be good news, but the wrinkles got me worrying.

It was a little bit strange that the job offer came without my even having applied. And, I have to admit I'd never even *heard* of the "Institute for Mathematical Analysis and Quantum Chemistry". This research center had no Web presence at all.

Perhaps they do government work that is so secret that security precautions prevent them from having a Webpage.

Perhaps the one-sided smile meant that he was saddened by the thought that I would be leaving soon.

Perhaps my theory about the symmetry breaking was just plain wrong.

Or perhaps things were about to take a turn for the worse.

Mulling over these possibilities as the train pulled into Ann Arbor. I started pulling my old green duffel from the overhead rack before we had come to a complete stop. So, when the train finally did stop, the cumbersome bag made me lose my balance. I fell on a woman talking on her cell phone. Her eyes narrowed, as if I'd tried to steal her purse, but she continued talking as if nothing had happened.

I was still apologizing as we entered the terminal building, but I was pulled away from her by a small man tugging on my shirt sleeve who meekly asked "Are you Igor, then?" Raising my hand one more time to try get the attention of the woman as she headed for the bathroom still chatting away, I recognized the futility of my desire to be forgiven and turned toward the man. My duffel slipped off of my shoulder and fell to the ground, sounding more like a dead body than the cheap clothes and toiletries it actually held.

"Yes", I said holding out my free hand, "and you are?"

"I'm Doctor Stein, from the Institute."

Well! The director himself had come to the station to pick me up. That seemed promising. My first impression of the man was a good one as well. His brown plaid shirt clashed with the blue slacks he wore, but he looked friendly. I am not sure exactly what gave me the immediate sense that the man was a genius, but I was already convinced that Stein was brilliant.

"You can call me Frank," he continued, shaking my hand. "Can I help you with the bag?"

I suspected that the bag weighed twice as much as he did, so I carried it to his green station wagon myself, and we were off.

At first, we merely gossiped about Rick. The two of them had apparently met in grad school and only barely stayed in touch since. For instance, he didn't know that Rick had

married the well known number theorist Lisa Rojahn, or that he had been in a water skiing accident and confined to a wheel chair for almost a year.

We also talked about my name, Igor Stravinsky. Everyone asks about it eventually. I began the usual spiel about the famous composer being a distant relative of mine and how my parents, being very fond of music, named me in his honor.

But, as we entered the city of Ypsilanti, Michigan, I began to ask more pertinent questions about my situation.

“So, are you taking me to my apartment first? Will I be living in Ypsilanti and commuting to...?”

“No, no,” he said cheerfully. “You’ll be living at the Institute with me, and it’s *in Ypsi*.” Then, seeing that I seemed concerned about this bit of geography, he added: “Don’t let that worry you, my boy. Did you know that MSRI isn’t in Berkeley but in Oakland?”

Somewhat comforted, I pressed on with my questions. “I did try to read some quantum chemistry papers, you know. There was this cool one where they were trying to predict the color of gold just from the mathematics of it. The funny thing was that because the nucleus is so heavy, they got it all wrong unless they included some relativity in there. Well, I understood that much. But, the details lost me. I don’t get Schrödinger equations and all that stuff about particles, yet. How much of that are you expecting me to learn?”

“Particles? You don’t need to be concerning yourself with that. I can do all of the chemistry. You will be my ‘hired gun’, handling the Riemann-Hilbert problems that pop up.”

“I didn’t see any Riemann-Hilbert problems in what I read. It’s too bad, too, because then maybe I would have understood it. So, could you explain to me how...”

“As I said, Igor, don’t worry about it!”

Finally, I asked a simple question. “Do all of the Institute’s employees live there, or just me and you?”

“Yes,” he said, and crow’s feet appeared by his right eye. There may also have been some by his left eye, but I couldn’t see them.

“I’m sorry, I guess you didn’t hear me. I asked whether you and I alone will live at the Institute or whether all...”

“I heard you. I heard you. Now, show me you’re as smart as Rick says and tell me why I said simply ‘yes’.”

It didn’t take me long to figure it out, but shock kept me quiet for too long and Frank began to hum the annoying music from the final round of Jeopardy. He seemed to be enjoying himself.

“Okay,” I said with too much anger. “I get it! It’s not much of an *institute* is it, if we’re the only two people there.”

“It’s significantly better than it was last week. One hundred percent increase in personnel! Not bad, not bad. And getting better all of the time. Aha! And here we are.”

I had not been paying close attention to where we were going during the last bit of conversation, and so I was surprised now to see that we were in a residential neighborhood that had clearly seen better days. The houses were large, with fancy woodwork and reasonably good wooden siding. But, they were all in serious need of paint, and grass growing in the many cracks made the sidewalk almost invisible.

We had stopped in front of one of the better looking ones, the weeds having been trimmed back enough that you could see the steps. It had been painted some time in the last ten years. Near the porch steps was a little sign that read “Mathematical Analysis / Quantum Chemistry” in gold letters on a black background.

That pretty much confirmed my theory about Rick’s smiles.

My area of mathematical expertise is solving Riemann-Hilbert problems. I like to think of myself as a wild animal trainer, but rather than making lions jump through hoops, complex-valued functions on a Riemann sphere do the jumps for me. Calculus students never see these animals; the functions in their zoo are well behaved, continuous and differentiable everywhere. But, in many applications, you need functions that jump in rather specific ways at prescribed locations.

The thing is, it can be pretty difficult to make the functions dance the way you want them to, especially when the moves get fancy. Using the stuff that *most* mathematicians know, it would be a nearly impossible task. But, I’m lucky that Rick knows the latest mathematical gimmick, a toolbox called “non-standard analysis” that gives me numbers infinitely smaller and infinitely larger than the usual ones. We use these as treats to coax the functions to do what we want. It’s still a lot of work, but it is straightforward enough, and not nearly as dangerous as lion taming!

My job at the Institute consists of using these techniques to solve the Riemann-Hilbert problems that Stein tosses at me. He carefully describes the jump he wants, where he wants it, and the *boundary conditions* that cage up the wild function away from the jump so it is always under control.

After I had been doing this for a few days, Rick called me to see how I was doing.

“This job isn’t so bad after all,” I told him. “The problem I worked on yesterday was really cool. You should have seen the monodromy that I ended up with! At first I couldn’t think of what I could do until I used a Möbius transformation and it all fell into place. You know?”

“Yes, I know.” Rick sighed and paused. “Has he talked to you about, uh, chemistry?”

“No. He told me not to worry about that side of it.”

“That’s good, I suppose. Well, keep in touch.”

As I hung up the phone, I had to remind myself of where I was in my latest computation. It was essentially done. So, I finished up what I was doing by writing a brief note to Stein:

Remember, in the last formula you ve got to think of the jump matrix as being an operator on the nonstandard Hilbert space H . Then, using your curve C we can define χ as:

$$\chi(\lambda) = I - \int_C F(\lambda) G(\mu) / (\lambda - \mu) d\mu$$

I’m not sure why Rick worried about me. I was beginning to have a great time. Living with Dr. Stein in the old house may have been rather unusual, but since he didn’t care what I did with my time off it wasn’t like living at home with mom and dad.

In any case, Stein approved of my hanging out on the University of Michigan campus with the math grad students. I attended their colloquia and seminars. I joined them in the U Club for beers. And I enjoyed a picnic lunch on the Diag with one student in particular, with high hopes of seeing more of her.

Dr. Stein (I still cannot quite bring myself to call him “Frank”) wanted me to stay connected to the mathematical community. He also wanted me to report back anything that anyone said about him. And I did. I told him that his early work in mathematical physics was apparently still respected, but that he is now considered quite the crackpot. I told him that the department was trying to figure out how to fire him despite his tenure. And I told him that people were very curious to know what we were doing at the Institute.

Since he had not sworn me to secrecy, I was comfortable passing information back the other way as well. I told my friends in Ann Arbor that Dr. Stein was definitely not the craziest person I’d ever met. As far as work goes, I told them he would frequently give me specific Riemann-Hilbert problems to solve, and he was always very grateful and positive when found solutions quickly. I had no idea what he was doing with my solutions or what it had to do with chemistry.

On our first official date, Becka and I caught a showing of “How to Steal a Million” at the Michigan Theater. We had seats right up front by the pipe organ, and in the scene

where Audrey Hepburn and Peter O'Toole are locked together in the closet at the art museum, her knee rubbed gently against mine and it stayed there.

Everything was going great. But later, over coffee, she started asking me about my job again. She asked the same questions she had asked at our picnic lunch the month before. I gave her the same answers. Why I should care about Franklin Stein and his micro-institute when all I could think about was her? She didn't agree.

"It looks nice," Dr. Stein was saying. "Yes, very nice. I think we're getting quite close to our goal now, boy. It won't be long now until they see that I'm not such a 'crackpot' after all! Just one question, are the branch cuts double ramified along the..."

I was barely listening. Instead, I was looking over his shoulder at the paper he had been working on. Below some complex analysis that I recognized, large and bold, I saw the expression:

$$\mathbb{F} \odot + O(\sigma) \leftrightarrow \mathbb{H} + O(\rho)$$

I was thinking, "What is this, *astrology*?"

"What *is* our goal, Frank?" I found myself shouting. I had never taken him up on his offer to call him by his first name before, so he knew right away that I was annoyed.

"I will tell you, son." He spoke calmly, in contrast to my agitated state. "You seem pretty bright and may be able to understand the importance of this work more than most of them. But first, are you sure you want to know?"

Ignoring his polite response, I continued yelling things I hadn't realized I had been thinking. "You know, I took classes in quantum physics and basic chemistry in college. I read a few papers on quantum chemistry before coming here. And, I never saw *any* Riemann-Hilbert problems in any of it. Are you even doing anything with the answers I'm giving you, or do you just go off and come up with another problem for me to work on? Is that what all of this is about, just wasting my time?"

"We are not wasting anyone's time. We are doing extremely important work here, Igor. The course of history will change." His voice became deeper and very nearly *boomed* as he said "After these many hundreds of years, Isaac Newton's dream will become a reality. By piecing together a bit of this field with a smidgen of that theory, we are resurrecting the greatest scientific achievement of all time, which died a slow and painful death from neglect and misunderstanding."

He would have gone on with this declamation if I hadn't butted in sarcastically with "What? Did Isaac Newton do *quantum chemistry*?"

“In fact, he did. Yes, Isaac Newton invented quantum chemistry as far as I’m concerned. Of course, he would not have called it that.”

I waited for him to continue, but apparently it was my turn to say something. So, after a reasonable silence, I went ahead and set him up as he wanted. With obvious skepticism, I asked “Alright, you win: What would Newton have called it?”

“*Alchemy!*”

It was rude to walk out on Dr. Stein without a word, but I needed some time to think.

Alchemy, huh? That’s what we’re doing at the institute: turning lead into gold! I couldn’t tell my friends at the University about that, and I suspected that it would not look good on a CV. Could I get hired, I wondered, at a real math department after spending a year doing witchcraft?

Mad and worried, I turned around and walked out of there. But, I had grown to trust Steina little bit, and to like him as well. So, I wanted to at least consider the possibility that he knew what he was doing.

That’s why I didn’t immediately get back on a train to home but instead went to the Grad Library and did some reading about alchemy.

I was surprised to learn that Isaac Newton actually *had* studied alchemy! (One point for Stein.) I had too much respect for Newton’s mathematical discoveries to discount him as another complete wacko. On the other hand, everything else reinforced my impression of alchemy as an early pseudo-science based more on wishful thinking than scientific rigor. The experts agreed that people who believed in alchemy back in the 17th century were basically crackpots, and anyone who still believes it today is plain nuts. (By my reckoning that made it two to one against poor Dr. Stein.)

I intended to pack my bags and head fro home, but I saw the light on in Dr. Stein’s room and knocked lightly on the door.

He seemed to know that I was planning to leave.

“Don’t go now, Igor. I’m... *We re* so close!” he pleaded.

“You can do it without me, Dr. Stein. You wouldn’t want me around anyway if I don’t believe, would you?” I thought that perhaps, like psychic powers and homeopathy, alchemy could be something that never seems to work when a skeptic is around.

“I can do almost all of it without you, and have been for years. But I never could understand that non-standard analysis that you do.”

People are always saying that to me; I don't understand why. "It's easy," I assured him. "Just pretend that there are real numbers that are infinitely big or infinitely small and do what you always do in a calculation."

"It is easy for you, and that is a gift. I would be ever so grateful, if you would share that gift with me just *one* more time. I think I've got it now. Solve this problem for me and I will show you that I do know what I'm doing."

Pity is not one of my favorite emotions, and I resented him for playing on it. But, his ploy worked. I stayed awake that night deriving complex functions with prescribed jumps for my boss one last time.

I probably was not looking my best when I wandered into the kitchen at 6:30 in the morning where I found Stein waiting for me with a fresh pot of coffee. As I struggled to stay awake and dunked my donut, he tried to explain his idea.

"What did Newton do immediately before he turned to alchemy?" he asked rhetorically. "He defended Gassendi's particulate description of light by showing that gravitational effects create a sort of wave-particle duality. But, then Newton had an idea that nobody else would think of for hundreds of years: what if all matter and not just light could be wavelike in nature. In modern terminology we would say that he had a soliton model of fermions induced by the nonlinear effects of gravity coupled to the Hamiltonian so that..."

"You're saying he found a theory of quantum gravity?" I said between sips. "A 'Theory of Everything', like physicists are trying to discover today?"

"Yes, but he had quite an advantage because he didn't spend hundreds of years believing in nonsense first."

"What nonsense would that be?" I inquired, thinking for the first time, that Dr. Stein was as unhinged as my friends said he was.

"Particles, my boy! The idea that matter is made up of particles is a terrible mistake our physicists made early on and we've been paying a price for it ever since."

"Of course matter is made of particles," I said patronizingly, though I was well aware that my limited background in college physics was nothing compared to his years of experience at the forefront. "I think we're pretty sure of that, anyway. Molecules, atoms, electrons and protons. I learned all about that in school and I've heard friends talking about things like quarks and moo-ons. Particles are an established fact, not nonsense."

"You mean *muons*. Yes, of course. I learned about all that, too. I learned the standard model. I even proved a theorem about super-symmetry that particle physicists are always making use of, but that doesn't mean it is an accurate description of the real world. I also learned that Columbus discovered America, but by the time you went to school they knew that it was not quite true. Here, let's try this. You say you took a

course in quantum mechanics, right? What did they tell you about the speed and position of a particle?"

"That you can't measure them both at the same time."

"Yes, but there is more to it than that. Physicists will tell you it is not only that the particle has a speed and a position that you do not know, the particle does not even *have* a position or a speed until you measure it."

"Yeah, I suppose I've heard that," I said, beginning to wonder whether all physicists are nuts. "That never quite made sense to me."

"And I'll tell you a simple way to solve the dilemma: get rid of the particles. There are no particles after all, only waves. The particle is just a figment of our imagination, and that is why it only has the properties we know it to have. *This* is what Isaac Newton realized when he reconsidered his theory of *Opticks*: all of this around us is a big wave, rippling and moving under the force of gravity. The gravitational effect may make it look like there are particles, but looks can be deceiving, no?"

To demonstrate this last remark, he held the eraser of his pencil loosely between his fingers and shook it up and down quickly. It sure looked like it was flopping around like a rubber tube instead of a wooden pencil, and I had to laugh.

"But," I said, almost starting to believe him, "if Newton knew all of this, why didn't he tell anyone?"

"He tried! That's what his *fluxions* were, a new mathematical notation for the wave nature of reality itself. If only he'd had your non-standard analysis to make it rigorous people might have seen what he was getting at. But, as it was, they thought it was an inferior attempt at defining derivatives and chose to go with Leibniz instead. But he wasn't only talking about functions and calculus, he was talking about the ultimate description of reality. That's what his alchemy was about."

"Okay, so now we get to it! What does it have to do with alchemy?"

"That's what alchemy *is!* Look, today's scientists wouldn't try to turn lead into gold. For them, particles are *particles*, unchanging by their very nature. But, suppose instead that lead and gold are different ripple patterns in a wave. With the right sort of nudging, you *can* change one into the other. I'm not just talking about a theoretical possibility. I know exactly what I need to do now, and I'm about to do it!"

"You mean, you're *really* trying to turn lead into gold?!? Come on, even you can do it, isn't there something more useful you could be doing with this new science of yours?"

"Sure, sure. If today's experiment works, I'll move on to something bigger: producing the *philosopher's stone*. The ultimate, universal substance, entirely unknown to modern science, a quantum superposition of the resonance patterns of all of the other elements.

Newton proved its existence mathematically but was never able to work out how to make it. All of the alchemists of his day made this their highest goal. The associated Riemann-Hilbert problem might be so hard that even *you* would have trouble solving it...but that's for another day. Today, we need to turn lead into gold because *that* is what it takes to get the attention of the media.”

“Well, there I guess I can agree with you, Professor Stein. If today's experiment works and you turn lead into gold, you could have a TV news crew here tomorrow and...”

“But, they are already here! And, today's experiment *will* work. How can you have any doubt? You solved the latest jump problem, no? Well, then we're ready to go!”

It was only then that I the light coming from the living room. I had mistakenly believed that it was sunlight, but this time of year it is not so bright so early in the morning. When I followed the professor through the doorway and my eyes became accustomed to the light, I saw several reporters and camera crews set up there. Dr. Stein went over to a computer terminal in the corner and began typing in the results of my night's computations.

The tall man with the huge pile of light brown hair and unbearably white teeth started talking as soon as we entered. “This is Tom Cannon reporting from the home of Dr. Franklin Stein, a U of M mathematics professor who is either a genius or a certifiable kook depending on who you want to believe. We are here today because Dr. Stein claims to be able to turn lead into gold using an old science called ‘alchemy’...”

Meanwhile, the Asian woman in the short skirt was saying “The mathematics department has refused to comment, but Peter Watkins, author of last year's best selling ‘The Alien Abduction Diet Plan’ insists that Stein's approach is well grounded and sure to produce spectacular results...”

And, most distressingly, the local access channel's young volunteer reporters were doing a terrible job of discussing me and my role in today's spectacular demonstration: “According to Stein, Igor Stravinsky -- no relation to the famous violinist -- played a key role in his research by solving Rhymon-Dilbert problems, math questions about integrals like those you might have seen in your calculus class...”

While I tried to hide in the corner, all eyes (and cameras) were on Dr. Stein.

“Thank you all for coming here today to witness a tremendous advance in the human understanding of the universe.” At this point, he switched on one of those hokey electric spark machines that you see in old horror movies. It was sitting on the coffee table and began buzzing and zapping as the spark climbed higher and higher before returning suddenly to the bottom. “This journey began long, long ago. Al chemie, like al gebra, and even (ironically) al cohol, all owe their origins to that period in the history of Arabia when their scientists were at the forefront of discovery. However, they did not know enough about waves to take it through to its conclusion. This, like so many of their great discoveries was lost to time. Similarly, Isaac Newton who finally knew enough to

put together the main ideas did not know enough about his own invention, the infinitesimal calculus, to..."

"Hey Doc," called out one of the cameramen, demonstrating that despite their mock sincerity, the news crews did not have any respect for Stein as a man of science. "We've only got a little bit of tape left here. Can you get on with it and turn the lead into gold already so we can get out of here?"

A few others snickered, whether at the idea that we might actually see lead turned into gold or at the cameraman's rude behavior I don't know, but Stein complied politely, and dramatically. He picked up a grey, metallic bar from the coffee table and dropped it. It shattered the table top and fell to the floor with a loud *thunk*. "This is lead," he said matter-of-factly. "This is a generator of vibrations in the electro-magnetic field," he continued, indicating the spark-making thing. "I place the lead bar on this vibrating platform which, like the field generator, is controlled by a digital computer."

"What kind of computer is it, doc?"

"The kind does not matter. The point is that it vibrates the bar and the electromagnetic field according to the real and imaginary parts of Igor's solution of the Riemann-Hilbert problem respectively so that..."

Though I was already in the corner, behind the recliner, I tried even harder to hide at the mention of my name and hoped that Becka was not watching TV this morning. Then the bar began to shake very quickly, and the pattern of sparks became much more erratic. Space itself seemed to bend visibly. This ripple of space began at the bar but seemed to spread out in all directions, even passing through me -- a very strange sensation -- before disappearing through the walls.

I was impressed! Dr. Stein seemed to know what he was doing after all. Or so I thought until I looked at him. He looked quite puzzled and concerned. Soon, however, he looked up again at the camera crews and smiled, proudly displaying a shining bar of gold.

The reporters were temporarily speechless. This was not going to be the humorous "human interest" story they had expected. Everyone in the room, even Stein, was only beginning to realize the implications of what we had seen. But, this sense of triumph did not last long.

Beginning with a few drops, soon a steady stream of water was dripping down on us from the chandelier. We all looked up at it, still and silent, noticing the cracks forming in the ceiling, before taking the necessary action. Everyone ran out onto the street as fast as we could. A few seconds later we heard the ceiling crash in and saw the living room flooding through the windows.

Up and down the street, every house was going through the same strange set of circumstances. Wet people ran out into the street as their homes and possessions were soaked in a flood of water.

“I don’t understand...” Stein whimpered. He looked miserable. But I was in a pretty good mood: I was actually part of something noteworthy.

“The pipes, Dr. Stein. These old buildings had lead pipes,” I explained. “You’ve turned them all to gold, which may earn the homeowners a nice profit in the long run, but gold is not strong enough to...”

“Of course, I know *that!*” he snapped. “But only the bar in our experiment should have been affected. The wave should have dissipated before ever reaching....unless...”

His expression vacillated between horror and anger as he grabbed me by the shoulders. “What boundary conditions did you use, Igor? The solution was compactly supported, wasn’t it?”

“Oh, well....that didn’t seem to be working out nicely,” I admitted, “so I used *periodic* boundary conditions this time. Was that, um, bad?”

Behind him I could see an angry mob heading in our direction, accompanied by the local access film crew. They did not seem grateful for our gift of golden plumbing.

“It depends,” he asked through a forced smile. “What do you think the effect will be on the world, on technology, on *the economy* now that every atom of lead on the planet has suddenly become gold?”

A slightly different version of this story was published in *Analog Science Fiction and Fact* (October 2007). The version you are reading is to appear in the collection **The Shape of Content**, an anthology of stories from the 2006 *BIRS Workshop on Creative Writing in Science and Math*.