SE Lie Theory Workshop 2021: Contributed Talks

Speaker: Tamanna Chatterjee
Affiliation: University of Georgia

Title: *Parity sheaves arising from graded Lie algebras*

Abstract: Let G be a complex, connected, reductive, algebraic group, and $\chi : \mathbb{C}^\times \to G$ be a fixed cocharacter that defines a grading on $\mathfrak{g}$, the Lie algebra of G. Let G_0 be the centralizer of $\chi(\mathbb{C}^\times)$. Here I will talk about G_0-equivariant parity sheaves on the n-graded piece, $\mathfrak{g}_n$. For the first half we will spend on derived category of equivariant perverse sheaves, bijection between the simple objects and some pairs that we are familiar with. In positive characteristic parity sheaves will play an important role.

I will define parabolic induction and restriction in graded setting. We will dive into the results of Lusztig in characteristic 0 in the graded setting. Under some assumptions on the field $\Bbbk$ and the group G we will recover some results of Lusztig in characteristic 0. These assumption together with Mautner's cleanness conjecture will play a vital role.

The main result is that every parity sheaf occurs as a direct summand of the parabolic induction of some cuspidal pair. Lusztig's work on $\mathbb{Z}$-graded Lie algebras is related to representations of affine Hecke algebras, so a long term goal of this work will be to interpret parity sheaves in the context of affine Hecke algebras.

Speaker: David Galban
Affiliation: University of Georgia

Title: *First and second cohomology groups for BBW Parabolics for Lie superalgebras*

Abstract: For semisimple Lie algebras, a well-known theorem of Kostant computes the cohomology groups of parabolic subalgebras, but it is unknown whether an analog of Kostant's theorem exists for Lie superalgebras. Seeking to provide the first calculations in this direction, in this talk, I will describe the cohomology groups for the subalgebra $\mathfrak{n}^+$ relative to the BBW parabolic subalgebras constructed by D. Grantcharov, N. Grantcharov, Nakano and Wu. These classical Lie superalgebras have a triangular decomposition $\mathfrak{g} = \mathfrak{n}^- + \mathfrak{f} + \mathfrak{n}^+$, where f is a detecting subalgebra as introduced by Boe, Kujawa and Nakano.

I will show that there exists a Hochschild-Serre spectral sequence that collapses for all infinite families of classical simple Lie superalgebras. Using this, I will provide examples of computation of the first and second cohomologies for some of the simpler cases.

Speaker: Ben Goodberry
Affiliation: Virginia Tech

Title: *Partially symmetric Macdonald polynomials*

Abstract: Symmetric Macdonald polynomials can be obtained by symmetrizing nonsymmetric Macdonald polynomials in all variables. It is also possible to symmetrize them in only a subset of the variables. Doing so produces polynomials with desirable properties including stability in increasing the number of symmetrized variables, a combinatorial integral form using diagram statistics, and Pieri formulas. We discuss a connection between these partially symmetric Macdonald polynomials and geometric objects introduced in the work of Carlsson, Gorsky, and Mellit.

Speaker: Nikolay Grantcharov
Affiliation: University of Chicago

Title: *BBW parabolics for simple classical Lie superalgebras*

Abstract: Classically, given a Borel subgroup B of a reductive algebraic group G, and a one-dimensional B-module L, the Bott-Borel-Weil (BBW) theorem computes the higher sheaf cohomology groups $R^j\mathrm{ind}_B^G(L)$. In the super setting, we introduce a parabolic subsupergroup B of a classical supergroup G which computes $R^j\mathrm{ind}_B^G(\mathbf{C})$. This result, combined with various properties of the parabolics subsupergroups, then verifies a conjecture of [Boe, Kujawa, Nakano] on the equality of various support varieties. This work is joint with D. Grantcharov, D. Nakano, and J. Wu.

Speaker: Shengnan Huang
Affiliation: Northeastern University

Title: *Root of unity quantum cluster algebras and Cayley-Hamilton algebras*

Abstract: The structures of maximal orders and Cayley-Hamilton algebras exist in many algebras within the framework of root of unity quantum cluster algebras. In this talk, I will show that the root of unity upper quantum cluster algebra is a maximal order, and the pair of it and its central subalgebra is a Cayley-Hamilton algebra. I will also discuss that all monomial subalgebras of root of unity quantum tori and any intersections of them over subsets of seeds are Cayley-Hamilton algebras. This is joint work with Thang T. Q. Le and Milen Yakimov.

Speaker: Dinkins Hunter
Affiliation: University of North Carolina, Chapel Hill

Title: *Stable envelopes of type A quiver varieties*

Abstract: Stable envelopes are canonical classes in the equivariant cohomology, K-theory, or elliptic cohomology of certain varieties that lie at the intersection of several different areas. They are important objects in physics, representation theory, enumerative geometry, and the combinatorics of symmetric functions. I will give a brief discussion of stable envelopes and some of their uses. Then I will present a combinatorial formula for elliptic stable envelopes of type A quiver varieties.

Speaker: Mee Seong Im
Affiliation: United States Naval Academy

Title: *Grothendieck rings of periplectic Lie superalgebras*

Abstract: The Grothendieck group is a fundamental invariant attached to an abelian category, which is defined to be the free abelian group on the objects of the category modulo the relation $[B] = [A] + [C]$ for every exact sequence $0 \to A \to B \to C \to 0$. I will explicitly describe the Grothendieck rings of finite-dimensional representations of the periplectic Lie superalgebras. This is joint with Shifra Reif and Vera Serganova.

Speaker: L. Andrew Jenkins
Affiliation: University of Georgia

Title: *The nilpotent cone for classical Lie superalgebras*

Abstract: Many aspects of the representation theory of a Lie algebra and its associated algebraic group are governed by the geometry of their nilpotent cone. In this talk, we will introduce an analogue of the nilpotent cone $\mathcal{N}$ for Lie superalgebras and show that for a simple classical Lie superalgebra the number of nilpotent orbits is finite. We will also show that the commuting variety $\mathcal{X}$ described by Duflo and Serganova, which has applications in the study of the finite dimensional representation theory of Lie superalgebras, is contained in $\mathcal{N}$. Consequently, the finiteness result on $\mathcal{N}$ generalizes and extends the work on the commuting variety. For the general linear Lie superalgebra $\mathfrak{gl}(m|n)$, we will also discuss more detailed geometric results of $\mathcal{N}$. In particular, we compute the dimensions of $\mathcal{N}$ and the centralizer of a nilpotent orbit, determine the irreducible components of $\mathcal{N}$, and show that $\mathcal{N}$ is a complete intersection. This is joint work with Daniel Nakano from the University of Georgia.

Speaker: Garrett Johnson
Affiliation: North Carolina Central University

Title: *On quantized nilradicals of parabolic subalgebras of simple Lie algebras*

Abstract: In this talk, I will discuss the automorphism group and other ring-theoretic properties of quantized nilradicals of parabolic subalgebras of complex simple Lie algebras. This is joint work with A. Jaramillo and H. Melikyan.

Speaker: Tekin Karadag
Affiliation: College of Charleston

Title: *Lie structure on Hopf algebra cohomology*

Abstract: It is known that the graded Lie bracket (Gerstenhaber bracket) structure on Hopf algebra cohomology of a quasitriangular algebra is abelian. We calculate the graded Lie bracket on Hochschild and Hopf algebra cohomologies of the Taft algebra T_p for any integer $p > 2$ which is a nonquasi-triangular Hopf algebra. We show that the bracket is indeed zero on Hopf algebra cohomology of T_p, as in all known quasi-triangular Hopf algebras. This example is the first known bracket computation for a nonquasi-triangular algebra.

Speaker: Scott Larson
Affiliation: University of Georgia

Title: *Small resolutions of closures of K-orbits in flag varieties*

Abstract: Let G be a connected complex reductive algebraic group and K the fixed points of an algebraic involution. The geometry of closures of K-orbits on the flag variety of G governs key properties in the representation theory of a corresponding real reductive group. We recall results from Barbasch-Evens on resolutions of singularities of K-orbit closures for the group $U(p, q)$. The group $Sp(2n, R)$ admits similar but more complicated resolutions, and we provide families of small resolutions in this case. We conclude with a general construction of resolutions of K-orbits.

Speaker: Jianping Pan
Affiliation: North Carolina St. University

Title: *Crystal for stable Grothendieck polynomials*

Abstract: Grothendieck polynomials are representatives of Schubert varieties in the K-theory of the flag manifold. Taking stable limit, we obtain symmetric functions called the stable Grothendieck polynomials. I will talk about a combinatorial construction of it, developed by Fomin and Kirillov, which turns out to have some nice crystal structure (directed, edge-labeled graphs from representation theory). There are also several properties associated with it, including some insertion algorithms. This talk is based on arXiv:1911.08732.

Speaker: Juan Villarreal
Affiliation: North Carolina St. University

Title: *Logarithmic vertex algebras*

Abstract: We introduce and study the notion of a logarithmic vertex algebra, which is a vertex algebra with logarithmic singularities in the operator product expansion of quantum fields; thus providing a rigorous formulation of the algebraic properties of quantum fields in logarithmic conformal field theory. We develop a framework that allows many results about vertex algebras to be extended to logarithmic vertex algebras, including in particular the Borcherds identity and Kac Existence Theorem. Several examples are investigated in detail, and they exhibit some unexpected new features that are peculiar to the logarithmic case. (This is a joint work with Bojko Bakalov)

Speaker: Zhang Weinan
Affiliation: University of Virginia

Title: *Drinfeld type presentations for affine i-quantum groups*

Abstract: The Drinfeld (loop) presentation has played a fundamental role in the representation theory of affine quantum groups . The i-quantum groups are coideal subalgebras of quantum groups arising from quantum symmetric pairs, and they are natural generalizations of quantum groups. Recently, Lu and Wang formulated a Drinfeld type presentation for affine i-quantum groups of split ADE types, and I generalized their work to split BCFG types. In this talk, I will present these constructions.

Speaker: Arik Wilbert
Affiliation: University of South Alabama

Title: *Real Springer fibers and odd arc algebras*

Abstract: Arc algebras were introduced by Khovanov in a successful attempt to lift the quantum sl2 Reshetikhin-Turaev invariant for tangles to a homological invariant. When restricted to knots and links, Khovanov's homology theory categorifies the Jones polynomial. Osvath-Rasmussen-Szabo discovered a different categorification of the Jones polynomial called odd Khovanov homology. Recently, Naisse-Putyra were able to extend odd Khovanov homology to tangles using so-called

odd arc algebras which were originally constructed by Naisse-Vaz. The goal of this talk is to discuss a geometric approach to understanding odd arc algebras and odd Khovanov homology using Springer fibers over the real numbers.

Speaker: Huanhuan Yu
Affiliation: University of North Carolina, Chapel Hill

Title: *Beilinson-Drinfeld Schubert varieties of parahoric group schemes and twisted global Demazure modules*

Abstract: Let $\mathcal{G}$ be certain parahoric Bruhat-Tits group scheme over the complex affine line. We develop an algebraic theory of twisted global Demazure modules, and prove that there exists a duality between the spaces of global sections of powers of the level one line bundle on the Beilinson-Drinfeld Schubert varieties of $\mathcal{G}$ and twisted global Demazure modules.