Laser gun battles, dangerous mutants, teleportation, androids, time travel. We are all familiar with these traditional science fiction motifs. But, who ever heard of mathematics in science fiction?!? Did the invading aliens in a B-movie ever threaten to "divide by zero" if the scientist didn't take them to his leader?

Mathematics in
Science Fiction

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I might define science fiction as fantasy fiction designed to appeal to those with a knowledge and interest in science. This definition addresses not only the fact, but the reason that science fiction fans like to read about pushing scientific topics just *a bit* farther than actual science can do now. In my view, the typical reader of science fiction is someone whose interest in the boundaries of scientific knowledge make traditional "realistic" fiction seem a little bit mundane. On the other hand, these same SF readers know too much science to be able to suspend disbelief for a traditional fantasy in which there are no constraints on the author. The fact that the author of a science fiction story cannot simply turn a prince into a frog—at least not without reference to some bogus "scientific" explanation involving a matter-transducing machine that controls the collapse of the quantum wavefunction—makes the story *more believable* to the reader.

Then, those of us who know and love mathematics can immediately see a tremendous potential for the use of math-ematics in science fiction. After all, the famous precision of mathematics makes it useful in making things convincing. (In fact, one might argue that a frequent *mis*use of mathematics is its inclusion in weak scientific papers for no purpose other than to make them seem more rigorous than they really are.) Moreover, those "crazy math nerds"—a group in which I proudly count myself—would most certainly find *fictional mathematics* as interesting as the fictional science of most science fiction.

As it turns out, the surprising thing is not this tremendous potential for mathematics in science fiction, but the fact that this potential has been largely *realized* without many people being aware of it. The "Mathematical Fiction Homepage" (see box below) is a comprehensive listing of significant appearances of mathematics in fiction. It presently lists 392 works of mathematical fiction—189 of which are categorized as science fiction—and is growing all the time.

Obviously, I could not hope to describe or analyze all 189 works of mathematical science fiction for you in this brief article. However, I would like to give you a quick "whirl-wind" tour, stopping at some of the most famous landmarks and pointing out just a few of my favorite lesser-known gems.

It seems natural to start this tour with *Somnium*. Written in 1634, this story about life on the moon is often referred to as the *first* science fiction story. Note that it is not the first story about a trip to the moon—Lucian of Samosata wrote about such trips more than a millenium earlier—but these earlier stories were not *science* fiction since neither the authors nor the readers knew enough science to make them so. On the other hand, the author of *Somnium* was none other than the famous mathematician Johannes Kepler. Kepler's understanding of planetary systems was intimately bound to his mathematical research, and it must be precisely for the reasons of interest and believability that he included pages of mathematical computation with the story.

Equations and mathematical notation have been thrown into more recent

The Mathematical Fiction Homepage is maintained online at

http://math.cofc.edu/kasman/MATHFICT/default.html

and seeks to be a comprehensive listing of all significant appearances of mathematics in fiction. The site lists movies, books, stories, TV shows and plays. Wherever possible, a link exists to works that are available for free on the internet or to sites at which the works can be purchased. Everyone interested in mathematics is invited to use this site to find works of fiction that they can learn from and enjoy. Moreover, anyone who knows any works of mathematical fiction not already listed is urged to contact the site administrators; the list continues to grow thanks to your help!

science fiction stories for similar reasons, though often by people who do not really understand their significance. A medical doctor named Miles Breuer wrote several mathematical fiction stories about travel to higher dimensional spaces. His 1929 *The Captured Cross-Section* begins with some vaguely reasonable mathematical notation that was probably learned in a Linear Algebra course but which is seriously misapplied and misinterpreted as having something to do with the theory of relativity. More reasonable computations of special relativity, even if very elementary ones, appear in Poul Anderson's *Tau Zero* (1970).

Perhaps the most famous example of mathematical science fiction is Edwin Abbot Abbott's 1884 masterpiece, *Flatland*. This book about a world of 2-dimensional polygonal creatures and their brief encounters with people from our 3-dimensional world was as much about Victorian society as it was an attempt to teach its readers about the mathematical concept of "dimension." However, its legacy has been its ability to convey an *understanding* of this abstract concept in a way that readers find both entertaining and enlightening. Perhaps this is why so many mathematical science fiction stories focus on the concept of dimension. Its direct progeny, such as Ian Stewart's recent *Flatterland* and Rudy Rucker's hilariously bizarre short story *Message Found in a Copy of Flatland,* are just a small sampling of the many science fiction stories that really address the mathematical aspects of dimension. After *Flatland*, Robert Heinlein's 1940 short story *And He Built a Crooked House*—about a house built in the shape of a tesseract—is probably the most famous, but Mark Clifton's *Star, Bright* is also notable for its inclusion of a topological twist—the Möbius Band.

Another rather well-known science fiction story with mathematical content is *The Feeling of Power* by Isaac Asimov. Although written in 1957 when 'calculators' were heavy mechanical devices based on complex gear systems, Asimov speculates about a future that might seem familiar to people in the early 21st century, when all students carry their own pocket-sized electronic calculators. The story concerns a future society in which all arithmetic is done by computers and people have completely forgotten how—and even *that*—it is possible for them to do it themselves. The hero of the story is a military officer who rediscovers for himself the ability to multiply numbers "by hand," a technique that is considered to be a new secret weapon by his superiors.

It is possible to encounter, and perhaps even to learn, some sophisticated and real mathematical results by reading science fiction stories. For instance, Martin Gardner's 1946 story *The No-Sided Professor* provides a fun introduction to topology, just as Stanislaw Lem's *The Extraordinary Hotel* (1968) introduces some of the interesting properties of countably infinite sets. More ambitiously, Greg Egan wrote a story called *The Infinite Assassin* (1991) whose conclusion can only be appreciated by mathematically sophisticated readers who understand what it means for the Cantor Set to be uncountable in cardinality and yet have measure zero!

On the other hand, it is perhaps more interesting to read some fictional mathematics—instances in which the mathematical results themselves come from the imagination of the author rather than from actual mathematical research. This must be very difficult to do, but some stunningly clever and well done instances of this motif do exist in literature. Greg Egan's novel *Distress* (1995) takes place at a realistically described mathematical physics conference where researchers are on the verge of completing a long sought after "Theory of Everything." Ted Chiang's 1991 short story *Division by Zero* contains no other hallmarks of science fiction, but explores the discovery of a proof that mathematics is inconsistent—something that *of course* has not yet been found, but whose existence cannot be ruled out according to Gödel's incompleteness theorem. As far as I am concerned, the most moving and clever bit of fictional mathematics is the conclusion of Carl Sagan's novel *Contact*. Although mathematics makes brief appearances in the film, the mathematical conclusion of the book was completely dropped from the movie. In the book, the main character finds a hidden message not sent by radio waves through space, but hidden in the very fabric of mathematics itself. Whether such a "message" exists and what it would mean if it did raises very interesting philosophical questions.

This idea from *Contact* that we might be able to communicate with aliens through the universal language of mathematics is a very common one in science fiction. I could list many stories, books and movies in which mathematics is used in this way. However, it might be more interesting for me to mention two "exceptions to the rule." Mathematician David Ruelle wrote an unusual bit of science fiction, published only in a book of serious essays by famous mathematicians, called *Conversations on Mathematics with a Visitor from Outer Space* (1998) which argues that *our* mathematics is essentially human in nature, and that aliens could be expected to have a different mathematics entirely. By concidence, in the same year Ted Chiang wrote a story that seems to demonstrate an example of this. Chiang's *Story of Your Life* (1998) involves the difficulty in communicating with an alien species whose mathematics, unlike our algebra-based mathematics, is based on variational formulations such as Fermat's Principle of Optics.

Being a mathematician myself, I appreciate these stories that build upon and utilize real mathematical knowledge. However, another side of mathematical science fiction that I have grown

to enjoy does not require the reader to really understand any mathematics at all, but instead to appreciate it as a sort of poetry. Consider Kathryn Cramer's eerie short story *Forbidden Knowledge,* which is built up around an actual quote from a mathematics book by Irving Kaplansky that seems to have a sort of dadaist beauty to it separate from any mathematical significance:

> We shall have no need to assign meaning to *Ext* itself; we shall speak only of its vanishing.

A more humorous approach to mathematical poetry is taken by Norman Kagan in his clever story *The Mathenauts*, which develops the notion that one can travel through space merely by thinking about abstract mathematics. It contains passages such as

> I was looking over Ephraim Cohen's latest paper, *Nymphomaniac Nested Complexes with Rossian Irrelevancies* (old Ice Cream Cohen loves sexy titles), when the trouble started. We'd abstracted...were ready for the first tests. I made the *Dold* invariant and shoved off through one of the passages that linked the isomorphomechanism and the lab.

Finally, I could not discuss the use of mathematics as poetry in science fiction without mentioning Eliot Fintushel, whose bizarre stories are regularly published in monthly science fiction magazines to wide acclaim despite their frequent (and bizarre) references to obscure mathematical topics such as the Banach-Tarski paradox. In an e-mail message to me, Fintushel summarized his story *Herbrand's Conjecture and the White Sox Scandal* by saying

> The idea is that the mathematical logician Jacques Herbrand who actually did die in a mountaineering accident the same time that his article was coming out on matters related to the Gödel consistency theorem, was murdered by aliens who needed to hush him up, because they were using certain lacunae in the metamathematical structures of mind or of space-time to sneak through in order to devour humanity. A psychic named Izzy Molson gets wise to them and fights them. In the course of this fight, the value of π changes, wreaking havoc on earthly geometry.

Another aspect of mathematics not directly related to its content is also important to its use in fiction, and that is the *stereotype* of mathematicians. On the one hand, there is the somewhat positive stereotype—one that I cannot say I mind terribly much—which says that mathematicians are very intelligent. Thus, it is easy for a writer to get across the fact that one of the characters is supposed to be very smart...they simply make him or her a mathematician. One example might be the mathematician character played by Jeff Goldblum in the film version of *Jurassic Park*. On the other hand, mathematicians also have a reputation—completely undeserved in my experience—of being cold and heartless. Many mathematical villians in books and stories display this supposedly evil side of mathematics. It must be admitted, however, that mathematics does not

change to suit the needs and desires of humans. This aspect of mathematics is well-utilized in the classic science fiction story *The Cold Equations* (1954) by Tom Godwin and in Yevgeny Zamyatin's 1924 dystopian novel *We*.

Such a "cold" view of mathematics, that it is frigid and uninteresting, is widely held in America today. One of the most exiting possible roles for mathematics in fiction is to change this viewpoint. In fact, Rudy Rucker's tongue-in-cheek short story *A New Golden Age* considers the consequences of a machine that could do just that; it can allow non-mathematicians to see and appreciate mathematics as mathematicians do. My own appreciation for mathematics was probably influenced by reading Stanislaw Lem's 1961 science fiction novel *Return from the Stars*. This book has several pages of mathematical dialogue that lies somewhere between the genres of "fictional mathematics" and "mathematics as poetry" described above. But, most moving to me was a description by an astronaut of why he became interested in mathematics during an incredibly long journey:

> I only came to understand this there. Because mathematics stands above everything. The works of Abel and Kronecker are as good today as they were four hundred years ago, and it will always be so. New roads arise, but the old ones lead on. They do not become overgrown. There...there you have eternity. Only mathematics does not fear it. Up there, I understood how final it is. And strong. ■