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by Juan de Burgos Román

(Translated from Spanish into English by Octavio Alberto Agustín Aquino with Alex Kasman)

Excellent Mister Counselor of Education and Culture
Excellent and Magnificent Rector
Excellent and Illustrious Ladies and Gentlemen
Ladies and Gentlemen

Preamble

To begin with, and even if it is not the best beginning, I must resort to your benevolence and beg for your forgiveness if this inaugural lecture does not become what you might expect from it. Because, aside from my incompetence regarding this art, you should not forget that I am a newcomer to it. And, from my natural clumsiness, it follows that I come here to talk you about something that deviates, albeit imprudently, from the accustomed in an academic ceremony like this one. Let me elaborate.

Not long ago, I had the unexpected fortune to run into a file that contained peculiar manuscripts, now very old, by Gudor Ben Jusá, a no less peculiar personage of former times who, as I later realized, is beginning to attract the attention of historians. At first, I had reasons to suspect that such a thing was a hoax, but after consulting knowledgeable people I can now guarantee the authenticity of these documents. It turns out that, considering what I have been able to find out, amazing facts are narrated in Gudor Ben Jusá's accounts, befallen long ago in a land far far away. In them, mathematics play an outstanding rôle, since it is used as a tool to solve the most diverse business or to explain what is happening there.

Our Rector, who had timely news about this event, hurried to lend me this podium so I can use it to divulge, obediently, the contents of this happy finding. I do not need to say that I accepted this honorable and opportune offer. But the Rector rushed to invite me, and I rushed to accept the invitation, since the manuscripts are reluctant to be interpreted and their translation into our language has been delayed more than expected. It was so extreme that I was forced to change the title of this address, and I apologize for this, since I thought that, in principle, Gudor Ben Jusá was also concerned about teaching, something that he seldom does.

I have translated three tales already; in all of them we are introduced to a prince, leader or chief who governs despotically, who did whatever he wanted, thus causing such a great fear that Gudor Ben Jusá disguises the facts changing from one narration to other not only the name of the despot, but also his way of exerting his government and, in general, everything around him. He even wrote the names of people and places backwards, from right to left. Believe me, I am very sorry I can give to you, such a learned audience, no more than these first three tales that seem to be the less substantial; there is time to know the others, which my prying had led me to believe use mathematical concepts of greater importance. The three tales go as follows.

First Tale

*The elliptic trick of a flatterer,
the hyperbolic response of a joker,
and the parabolic finale of all this.*

Once upon a time, a long time ago, in a land far far away, there was a whimsical monarch, arbitrary and not exactly intelligent. We are talking about Sojotna XIII. Our story begins the day the king ordered the Great Duke Atsiboc, his prime minister, to dig a hole for a lagoon in a site called Onipotnik. This place was located amidst a plateau, a half mile away from the plentiful river Ulg-Ulg, which flowed peacefully around there.

The Great Duke ordered to dig a pit in the marked place. He water-proofed it using the best of his knowledge and filled it with water. The water evaporated a short time later. When, surprisingly, Sojotna XIII went to the lake prepared for him, he found an empty and dry pit. The king burst out in rage and told to the Great Duke that, unless he wanted to be executed, he

must turn Onipotnik into an authentic eden; there must be enough water for high and splendid poplars to grow, like those covering the banks of Ulg-Ulg river.

The prime minister hastened to do anything necessary to fulfill his king's desire. The king, worried about other whims, seemed to forget about the lagoon.

Some time later, Sojotna XIII asked, incidentally and as if it were unimportant, for the poplars of Onipotnik. The Great Duke, in order to please the king and believing he had lost any interest in the matter, committed the imprudence of claiming that the trees had grown twice the height of those in the banks of the river. Then Sojotna XIII, with a malicious gesture, manifested his wishes of checking it and ordered a journey between the lagoon and the river so he can compare the poplars from that place. The prime minister, finding himself up a creek without a paddle, sought the help of Artemóeg, an odd character, that knew how to find solutions for those problems that nobody else could.

Artemóeg began noting that, in order to see two equal poplars one greater than the other, it suffices to place oneself closer to the first than to the second. He said that, if two poplars are seen away enough, such that one of them looked twice as high as the other, one must position oneself approximately where the distance from the first is the half from the second. Thus, the royal retinue must follow the path that results from joining the points that lie half the distance from the lagoon than from the river. In Artemóeg words, this path is called an ellipse. It is said that it is the ellipse which has a focus in the lagoon, the corresponding directrix in the river and eccentricity $\frac{1}{2}$ (the ratio of the distance between the foci and the directrix).

The prime minister thought this elliptical path solved his problem. But, for Great Duke's misfortune, prince Otirbak heard about this plot and prepared a trap. He kidnapped Artemóeg and forced him to change the path such that, while traveling through it, the lagoon's poplars looked half the size of those of the river, instead of twice. "The path you are asking for", said Artemóeg, "is very well approximated by the one formed by the points that lie twice the distance from the lagoon than from the river. In the words of those who deal with geometric stuff, I am talking about the hyperbola that has a focus in the lagoon, the directrix in the river and eccentricity 2. This hyperbola has one of its branches passing between the lagoon and the river. You should travel through this branch, my prince."

With such a hustle and bustle, spying one to the other, preparing mis-

leading paths, with such a fuss, the king ended finding out all the trickery among his prime minister and his heir. Infuriated, he jailed the Great Duke, scolded the prince (he was his child, after all, so he does not need to overdo it), and ordered Artemóeg to be brought into his presence. Artemóeg told him how all that mess caused by the paths worked. Sojotna XII commanded Artemóeg to prepare a third path, this time with no cheat. Artemóeg spoke and said: “My king and lord, the path where equal things look equal is the one that travels through the points equidistant from both things and, thus, you should circulate between the lagoon and the river following the curve, the so-called parabola, consisting in the points lying the same distance from the lagoon and the river.”

Sojotna XIII travel through the path specified by Artemóeg and could appreciate that the lagoon’s poplars and those in the banks of the river were of equal height. Afterwards, he called the Great Duke and told him that, his audacity being so great, he was forced to impose on him a severe punishment:

For a whole year, he must attend to the lectures delivered by Artemóeg about the so-called Geometry. He must be graded every two weeks and, whenever he failed (as it was expected) he must take summer courses. In case he interrupted the course, he would be executed. In the unlikely event that he passed, he must take, immediately, Geometry II, applying the same criteria as the previous one. He must continue successively until achieving his disappearance from the world of the living, in the hands of the executioner or because of Geometry.

Against all bets, the Great Duke passed, one after another, the Geometry courses. The king died the year when he was attending to Geometry VII. Sojotna XIII was succeeded in the crown by his son Otirbak, who forgave Atsiboc. Atsiboc, by those times, oddly enough, grow fond of Geometry (believe it or not!).

Some people say that from this happening follows, evidently, that it is not a good business to be a sycophant or a flatterer, since those one flatters or soft-soap end kicking the adulator and, later and beyond doubt, become our worst enemies.

Second Tale¹

*On the time when feudal lords
divided their serfs in halves, quarters
and even eight parts.*

The land of Aremihc was ruled long time ago by a feudal lord, called Enimsi Lla, who did whatever he willed. He found utterly difficult to recruit, among the common people, the serfs he required. And he needed a lot. He asked for the help of Rerecros, the great wizard, who got down to work immediately.

After brooding about it, and with the assistance of “The Great Book of Occult Knowledge and Magical Devices”, sketched out an odd artifact, which he called duplicator and which had one entrance hatch and another exit hatch. He expected to obtain, each time a serf were introduced inside the duplicator, two serfs out of it. But spells, no matter how magical, have drawbacks and, where there is little substance, you cannot pull out much. It turned out that things did not work as the wizard Rerecros expected. When he turned the duplicator on and introduced inside (forcibly) a serf named Nrutruoy, two servants came out. But they were tiny, half the original size; they were called “nrutrouyies”. They were equal and had the same craftsmanship of Nrutruoy, i. e., they keep the same proportions. Therefore, the duplicator was not such, but a “multireducer” instead. The common people called it the “Nmad machine”.

In regard of this, the feudal lord thought that many tasks could be perfectly accomplished by a little serf with the same abilities as a regular serf. So he decided to resort to such a machine to trade some of his serfs for twice the number of little serfs. He arranged to double the number of guards. He assumed that, for watching the battlements of his castle, the size of the watchman was unimportant.

For these guards, as for everybody, it was necessary to tailor special garments, to keep them from freezing to death in the high battlements. When the tailor prepared to make such cloaks, he supposed that each of the little guards required half the cloth usually needed for a regular guard, since they were half the size. But, surprisingly, he discovered that the cloak of a little

¹*Translator’s note: I decided to translate the puns here, to make them as funny as the original ones. (Remember to read the names backwards!)*

guard required only a quarter of the cloth needed for a guard.

Later, when he thought of it calmly, he realized that the little guards were not only half the normal height, but also their waists were half the normal length and half the normal width. Therefore the pieces of cloth now needed were half the regular length and half the regular width. It turned out that from the former pieces four new were obtained. In other words: when all the lengths are reduced in a half, the areas reduce to the fourth part.

Enimsi Lla, the lord, having heard of all this, thought that a from a regular serf not only two little serfs could be obtained, but four. Then he called the wizard, and told him his magic was flawed and he commanded him to apply himself to improve his machine such that, for each serf of input there were four little serfs of output, each of half size. Rereros eagerly devoted himself to the problem and, after staying awake several nights, he finally could satisfy his lord's desires. The lord prepared to use again the machine to enlarge the number of laborers on his service. Thus he succeeded in quadrupling his workforce, though now he had "little laborers". He trusted this would improve the profitability of his land.

But things were not as expected. Quite on contrary, that year the harvests reduced in a half. The feudal lord, after brooding a lot on it, figured out what happened: he learned that four little laborers ate half as much as a regular laborer and, thus, it seemed reasonable that four could not do but half the work of a regular laborer. He thought that the energy of the "little laborers" depended on their volume (stomach capacity) more than on their size or surface. Therefore, he decided to determine the volume of the little laborers. He discovered that, putting them in a vat full of water and measuring the quantity of spilled liquid, to equal the volume of a laborer eight little laborers were required, and not only four as he previously believed. He noted, in support of his discovery, that in order to determine the volume he must multiply height times length times width. If the three measurements were reduced by half, the volume was now the eighth part.

Aware of this, he called the wizard again ostensibly angry, and told him he was cheating with his machine, since its output was only half the quantity of the input: for each pound of serf delivered, only half a pound of little serf were returned. Without waiting for a reply, he imposed him a short deadline to achieve that, for each serf of input for the machine, there were eight little serfs half the size of output. The wizard employed all of his knowledge, all his esoteric books, his most effective spells, his most potent incantations, everything within his reach and, when the deadline was about to expire, he

accomplished what his lord demanded. Now the machine had no loss, it preserved weight and volume. It was like a meat grinder, where all the input became output.

Shortly after the happenings we have just narrated took place, a huge earthquake occurred, and nothing was left. Everybody died, from the feudal lord to the very last little serf, except a man called Enán. He was the man whose work was to guard the machine in the room where it was stored. Enán, entrapped in that place, discovered a crack in the wall and tried to escape through it, but the breach was too narrow. Then Enán got into the machine, from which eight dwarven² got out and exited through the crack. By the time the first seven were out (the famous dwarves from fairy tales), another earthquake reduced the hole to less than half a foot. The eighth dwarf got into the machine, a procedure that the resulting dwarfs repeated, until everybody fit in the little orifice. Now outside, they hid in the forest so well that nobody has seen them since. They are the gnomes.

Third Tale

*The sun, rising for everyone,
shines more intensely on
some people than on others.*

The hitherto powerful nation of Sorar elected annually their Dílada or Big Chief and they did it in a random way, by raffle, among the twelve chiefs that ruled the country. The process, surrounded by ritual and loaded with symbolism, seemed to be irrefragable. Nevertheless, the outcome was not as expected: some chiefs were elected much more frequently than others. Allegedly, the Sun god favoured some of them, altering the laws of chance. Gossip said, although, that such disruptions were caused by some mischievous men... you know there is no shortage of them. But we should not jump to conclusions, let's examine this rigorously.

The Sorar divided their realms in four provinces symbolized by the colors red, blue, white and green. Among the Sorar, twelve families ruled the country. The heads of these families were twelve chiefs that, under the command of the one they called Dílada, governed all provinces. Three aspects

²Translator's note: in Spanish, the diminutive of Enán is identical to the word for "little dwarf".

regarding the exercise of power concerned them particularly: to manage the provinces' civil government, to support an effective and powerful army and, lastly, to be meticulously aware about everything related to the cult of the divine, the Sun god. These three government affairs were represented by a scepter, a sword and a sun, respectively.

During summer solstice, a great celebration took place when the Sun was honoured with solemn festivities and, taking the opportunity, the people amused with big feasts, balls and soirées that, frequently, lead to authentic bacchanalia. During these festivities, the Sun god chose, from among the twelve provincial chiefs, the Dílada. The Dílada would be the supreme leader, boss and unquestionable guide of Sorar country during the next year. And the Sun god made use of chance to do this.

A bench was disposed for this purpose, with twelve seats, where the twelve chiefs sat. They did so in a random manner, at whim. Thus they signaled their fair proceedings. The bench was formed of four equal strips colored with red, blue, white and green paint, alluding the four provinces. In each of these stripes three seats were placed, one crowned by a scepter, one with a sword and the last with a sun. This platform was known as the “square of command”. Later, two hoods were prepared and a specific number of balls were placed in each. In the first they put four balls: one red, one blue, one white and one green. The second hood contained three balls, each one had a drawing of one of the power symbols (scepter, sword and sun). The Sun god placed his will in a virtuous maid, who had taken a good dose of certain hallucinogenic plants; once it has an effect, the maid blindly drew three balls from the provinces' hood and two balls from the power symbols' hood. Then, the Eldest of Elders, on behalf of all Council, let them know the Sun's will by showing the two remaining balls to the people, one from each hood. The chief sitting in the seat whose color and symbol matched the ones the Eldest of Elders exhibited in front of everybody was the chosen one, the Dílada of the Sorar people.

The Sun seem to have special predilection for some chiefs, who were successively chosen as Díladas. The god's bias was due, for sure, to the many accomplishments and courage of these leaders of Sorar people. It is known that a certain Rodamit was the chief of his province during 24 years and, in this period, he became Dílada ten times. The Sun's tilt toward him was so evident that it provoked a great envy. People began to say that he committed fraud to alter, in his favour, Sun's will. The non-believers did not accept that, among the 24 times he attended to the Dílada election, he was

chosen in 10 of them. As there were twelve candidates, the probability of becoming the Dílada each one had was $\frac{1}{12}$. They added that, since Rodamit attended 24 elections, he was expected to be chosen $24 \times \frac{1}{12} = 2$ times or, if he were lucky, even 3 of 4 but not 10 times.

Rodamit's case was so striking that the Council of Elders asked him, with the utmost respect, to manifest whether he knew any cause explaining how he became Dílada 10 times. Rodamit said that he had been aided by the Sun and, to do so, the god did the following. The election's day, while he was praying at dawn, facing east, as the ritual commanded, the Sun sent him the color that would be obtained during the Dílada's appointment. Rodamit seated in one of any of the three seats coloured as the Sun had pointed out for him. Thus, the probability of being chose was $\frac{1}{3}$. Since he attended 24 times to the elections, it was reasonable that the appointment fell on him about $24 \times \frac{1}{3} = 8$ times and, with some luck, it was not rare he had become Dílada 10 times.

Unwilling to refute the above and for truth's sake, it must be admitted that there were some clues that a man, on Rodamit's service, furtively changed one of the colored balls for a similar but heavier one. Therefore, this ball rested in the bottom of the hood and it was almost certain that, after the extraction of three of the four balls, the heaviest one remained. If this were true, Rodamit knew the color to be chosen by the Sun. But we ought to ignore these rumours since, surely, they are a figment of the envy and lack of any basis.

Rodamit's statements to the Council of Elders made some of them think about the meaning of the so-called "probability". Consider, they said, any of the past elections when Rodamit was appointed as Dílada. In it, for us he had a probability equal to $\frac{1}{12}$ of being elected. For him, this probability happen to be $\frac{1}{3}$. For the Sun, being omniscient, such probability was equal to 1. This stunned them a little, since the probability of an event turned out to be variable, changing from one person to another; it depended on the information available to them. This lead them to think that probability is a number that measures our greater or lesser ignorance about the possibility of certain event occurring and, without surpassing unity, it grows with our knowledge concerning what is about to befall.

Epilogue

The skill manifested by the people in the old days in the use of mathematics for solving and explaining the matters that concerned them, such a wit, has not decreased with the passing of time. Quite the contrary: there are lots of people who turn to them to solve the most diverse business. And when I say “turn to”, I am not referring only to those that use mathematics as a toolbox, where they take in each case and depending on the specific need the gadget that fits the most. I also speak, and mainly so, about those who, day by day, devise new tools to face up the questions that today, as yesterday, we may come up with.

The enormous complexity of these problems, progressively more intricate, that concern contemporary people, force those who create such tools (by fortune, we are plenty of this people at this University) to apply themselves with great imagination and talent. I deem it righteous to acknowledge the inventiveness of these creators of profitable tools and, in the future, to show the due gratitude to the tenacity they devote to this work. Without doubt, the University itself is the first beneficiary.