Key Ideas: Absolute Extrema

- In Calc I we answered absolute max/min problems like:

**Question 1:** Let \( g(t) = t^2 + 2t + 1 \). Find the absolute maximum and minimum value that the function \( g \) takes on the interval \(-2 \leq t \leq 2\).

- Recall that in the one-variable case, we were only really able to make general statements about locations of global extrema when we were considering a continuous function on a closed interval. (Otherwise, there was always the possibility that the function was increasing and increasing without ever reaching a maximum and this was difficult to identify in general.)

- Remember also that a function could have an absolute maximum at a point that is not a critical point, if it happens to be at one of the ends of the interval!

- To answer this question we do not have to use the second derivative test to identify local extrema. Instead, we use a method I like to compare to a “line-up” on police shows. The suspects are any critical points inside the interval and the endpoints. One of these must have the largest value and the other the smallest when plugged into the function. So, we just plug them in and compare.

- Well, the same is true here. We need some terms to be able to state the corresponding theorem: A **closed set** in the plane is one that contains all of its boundary points (points that are arbitrarily close to some points in the set). A **bounded set** is one that is contained within some (perhaps very large) disk.

**Extreme Value Theorem:** If \( f \) is continuous on a closed bounded set \( S \) in the plane then \( f \) attains an absolute maximum and an absolute minimum value somewhere in \( S \). To find them,

- Find the values of \( f \) at the critical points of \( f \) in \( S \),
- Find the extreme values of \( f \) on the boundary of \( S \) (by parametrizing it and reducing it to a question of one variable),
- Choose the largest and smallest of these, because they are the absolute maximum and minimum values.

**Question 2:** Find the absolute max and min of \( f(x, y) = x^2 + 2xy + y \) on the rectangle \(-2 \leq x \leq 2 \) and \( 0 \leq y \leq 1 \).

- Just as in the one variable case, it is not necessary to use the second derivative test if you are only trying to find the absolute max/min values on a closed bounded region. Doing so would be a waste of your time, and I do not think you will have much time to spare on the next test.
In the example above, the function and the restricted domain seem to come from nowhere. Of course, when this method is used in applications, the function to optimize and domain are determined by the situation. Here is an example in which the application is itself mathematical:

**Question 3:** Find three positive numbers whose sum is 100 and whose product is a maximum.

- Note that *next* time we will learn another method for optimization that is actually quite different from the one-variable case.

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**Homework**

**Section 14.7:** Read the section and do these problems: 29–31, 32, 44, 49–52, 54.

**About #54:** I find this an amusing vindication of my view of mathematics as compared to the famous mathematician GH Hardy. In his book *A Mathematician’s Apology*, Hardy brags that mathematics is completely *useless*. Nevertheless, number theory (his area of research) now forms the basis for the encryption on which internet security is based, and his theorem on genetics written with physician Wilhelm Weinberg (see question) is still frequently used by biologists around the world.