Key Ideas from Section 4.9: Antiderivatives (Cont’d)

• In general, finding antiderivatives is much harder than finding derivatives. (The shortcut rules we learned for differentiation are difficult or sometimes even impossible to apply in reverse.) In fact, there are simple sounding functions, like $f(x) = \sin(x^2)$ for the antiderivative doesn’t even have a formula that you can write.

• However, there are many situations in which one knows the derivative and needs to find the antiderivative. That is, even though it is harder to antiderivative, it is perhaps even more important than differentiation. We will spend much of the rest of the course talking about antiderivatives and their applications and this will also be the main topic in Calculus II.

• Graphically, the set of all antiderivatives of a function $f$ are a bunch of graphs that are the same except of a vertical shift. The simple explanation for this is that a vertical shift does not change the slope! (If the function is discontinuous, then you can shift each piece separately.)

• Do not mistake our inability to find a formula for an antiderivative with the notion that a function does not have one. As we will learn later in the course, every function $f(x)$ has infinitely many antiderivatives (on an open interval where it is continuous). It just may be that there is no formula for the antiderivative even if there is a formula for $f$.

• Even though finding a formula for an antiderivative might be impossible, we can always sketch the graph of an antiderivative using the ideas from curve sketching. The sign of $f$ tells whether its antiderivative is increasing or decreasing, and the sign of the slope of $f$ tells us whether its antiderivative is concave up or down. Similarly, a place where the graph of $f$ crosses the $x$-axis is a local extremum of the graph of its antiderivative and a local extremum of $f$ is an inflection point of the antiderivative!

Question 1: Sketch the antiderivative of $f(x) = \sin(2\pi x^2)$ whose graph goes through the origin. Try to find a formula for this antiderivative – but I assure you that you won’t find one that can be written in terms of the functions that we have learned.

• One application of antiderivative is the ability to determine the position of an object moving on a straight line given information about its velocity or acceleration. Given a formula $a(t)$ for the acceleration at time $t$, you can anti-differentiate to get the velocity at time $t$ (if you know the velocity at some time to determine the constant.) Similarly, if you know the velocity $v(t)$ you can antidifferentiate to get the position (if you know the position at some time).

✓ In fact, this is a very realistic application. There are many situations in which we know or have direct control over the acceleration of a moving object and yet are interested in its position. In the following two examples, a rocket provides the acceleration. (Perhaps they were inspired by my favorite scene in the film Apollo 13 where the astronaut has to use calculus to determine how long to fire the rocket in a real life-and-death situation.) For homework, you will answer a similar problem in which the known acceleration is provided by the brakes on a car.
**Question 2:** A rocket is launched so that its upward acceleration at time \(t\) is \(a(t) = 3 + 2t \text{ m/s}^2\) for the first 30 seconds. Given that it starts at height zero with velocity zero, find a formula for its position at time \(t\) during those first 30 seconds.

**Question 3:** The space shuttle Endeavor is about to dock with the station Mir and is travelling straight towards Mir at a constant velocity of 5 meters per second. Let us call the time that the shuttle is only 12 meters away \(t = 0\) seconds. Beginning at time \(t = 0\), it fires rockets which provide a constant acceleration of \(1 \text{ m/s}^2\) in the opposite direction. They will continue to fire these rockets (to slow the shuttle down) until the ships touch.

- Find a formula for the function \(s(t)\) which tells how far the shuttle has moved from time \(t = 0\) seconds.
- When do the ships touch? (That is, when is \(s(t) = 12\)?)
- At what speed is the shuttle moving when they first touch?

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**Homework**

**Web Problems:** Due on Friday.

**Written Problem:** Please do the following “curve sketch” problem for next Monday (11/15) to be turned in during class on paper.

The function \(f(x)\) is defined on the interval \(-2 \leq x \leq 4\). Use the graph of the function \(f(x)\) below to answer questions about its antiderivative \(F(x)\). [Note: \(F(x)\) is not the function I have drawn in the graph below. This is its derivative \(f(x) = F'(x)\).]

1. On what interval(s) is the antiderivative \(F(x)\) decreasing?
2. On what interval(s) is the antiderivative \(F(x)\) concave-down?
3. At what point(s) is the tangent line to the antiderivative \(F(x)\) horizontal?
4. Sketch two different antiderivatives of \(f(x)\):