Key Ideas from Sections 4.5 and 4.6

“Curve Sketching”:

- A good curve sketch is like a good caricature. It doesn’t have to be completely accurate (like a photograph). When a person looks at a curve sketch, they should be able to locate and identify all of the “key features” of the function. For this reason, you may have to over-emphasize some of these key features, just like a caricature artist might. Your calculator doesn’t know how to do that. It doesn’t know where to find the key features, but we do. We just learned that the local extrema occur where the first derivative changes sign and the inflection points occur where the second derivative changes sign. We can locate asymptotes (using limits) and emphasize them by drawing appropriate dashed lines on the sketch.

- In some senses, we are going to learn nothing new today. This is just an opportunity to “put it all together”. The suggested procedure for sketching the graph of a function is (see p. 308):
  - Determine the domain of the function.
  - Determine the intercepts.
  - (Symmetry: This can be a useful guide in curve sketching, but I will not be talking about it or asking you to do any problems that would require it.)
  - Find all vertical and horizontal asymptotes.
  - Determine intervals on which $f$ is increasing/decreasing.
  - Identify all local extrema.
  - Determine intervals on which $f$ is conc. up/down and location of infl. pts.
  - Combine all of this information into a curve sketch!

**Question 1:** Try sketching $y = \frac{2x^2}{x^2 - 1}$, $y = \frac{x^2}{\sqrt{x + 1}}$, and $y = (x - 1)e^x$.

- Sometimes, to save you trouble, the derivatives may be given so that you do not need to find them yourself:

**Question 2:** If $f(x) = (3x^2 - 4)/x^3$ then $f'(x) = 3(4 - x^2)/x^4$ and $f''(x) = 6(x^2 - 8)/x^5$. Use this information to make a complete sketch of $y = f(x)$.

- The book suggests that sometimes you will need more than one sketch to accurately convey the information about the graph. (See example 1 on page 315.)

- Use your calculator and your brain together. Use one to check the answer provided by the other. If you can’t solve $f'(x) = 0$ (or $f''(x) = 0$), try graphing $f'(x)$ (or $f''(x)$) and estimating the coordinate of its root with your calculator. But don’t trust your calculator over your brain. I could easily give you a misleading example which has a local maximum or minimum but looks like it doesn’t have any local extrema when graphed naively on the calculator.

**Question 3:** Use your calculator and calculus together to sketch the graph of $f(x) = 2x^6 + 3x^5 + 3x^3 - 2x^2$. 
Finally, sometimes all of the information is given to you and it is simply your task to put it together into a sketch. (This is an especially common type of final exam question.)

**Question 4:** Sketch a graph of a continuous function \( y = f(x) \) that satisfies the following:

\[
\begin{align*}
f(-2) &= -5 & f(2) &= 2 & \lim_{x \to \pm\infty} f(x) &= 1 \\
f'(2) &= 0 & f'(4) &= 0 & f'(-2) &\text{ Does Not Exist} \\
f''(2) &= 0 & f''(3) &= 0 & f''(5) &= 0 \\
f'(x) &> 0 \text{ for } x \in (-2, 2) \cup (2, 4) \\
f'(x) &< 0 \text{ for } x \in (-\infty, -2) \cup (4, \infty) \\
f''(x) &> 0 \text{ for } x \in (2, 3) \cup (5, \infty) \\
f''(x) &< 0 \text{ for } x \in (-\infty, -2) \cup (-2, 2) \cup (3, 5)
\end{align*}
\]

You should clearly indicate all local extrema, inflection points and asymptotes.

**Homework**

**Written Homework:** On Friday March 25th, turn in problems 18 and 44 from Section 4.5 (page 314). For full credit, you should show all the work of identifying intervals of increase/decrease, local extrema, intervals of concavity, inflection points, asymptotes, and put them together in a clearly labeled graph.

**Web Homework:** There are no Web Assign problems for this homework set.