**Key Ideas from Section 4.3**

*What do $f'$ and $f''$ tell us about the graph of $f(x)$?:* Increasing/Decreasing, Concave up/down, local extrema and inflection points.

- **The main idea:** we can say a lot about a function $f(x)$ and what its graph looks like just from information about its derivatives.

- **To determine where $f(x)$ is increasing and where it is decreasing:**
  1. On a number line mark each value where $f' = 0$ or DNE. (These marked points are the only places that $f$ can change its direction.)
  2. Determine the sign of $f'$ on each interval “cut” by the marked points. (One way to do this is to simply pick some number $c$ in that interval and check the sign of $f'(c)$.)
  3. On any interval where $f' > 0$, $f$ is increasing. On any interval where $f' < 0$, $f$ is decreasing.

- **The First Derivative Test for Local Extrema:** If $c$ is a critical number for $f$ and $f$ is continuous at $x = c$, check to see if $f'$ changes signs at $c$.
  
  $$
  + \rightarrow - \quad \Rightarrow \quad \text{local max at } x = c \\
  - \rightarrow + \quad \Rightarrow \quad \text{local min at } x = c \\
  \text{no sign change} \quad \Rightarrow \quad \text{not a local extremum}
  $$

- **Concavity:** “Concave up” means that the slopes of $f$ increase as you move right. This is the same as saying that $f'$ is increasing or that $f''$ is positive! “Concave down means that the slopes of $f$ are decreasing as you move to the right. This is the same as saying that $f'$ is decreasing or that $f'' < 0$. [Example: the graph of $y = x^2$ is concave up everywhere and the graph of $y = -x^2$ is concave down everywhere.]

- **Notice that concavity is exactly what we were looking at when trying to decide if the linear approximation is an overestimate! If $f''(a) < 0$ then the linear approximation near $x = a$ gives an overestimate.

- **A point on the graph where the concavity changes is called an inflection point.** You can find it (just like finding local extrema) by checking to see where $f''$ changes sign. (Note: an inflection point of $f$ is a local extremum of $f'$.)

- **Second Derivative Test:** If $c$ is a critical number for $f(x)$ and
  
  $- f''(c) > 0$ then $f$ has a local min at $x = c$  
  $- f''(c) < 0$ then $f$ has a local max at $x = c$

  Warning: if $f''(c) = 0$ or is undefined then you cannot conclude anything from this test.

- **Note:** if I tell you a formula for $f'$ or give you its graph, you can find the local extrema, inflection points, and the intervals on which $f$ is concave up/down, increasing/decreasing without having to know $f$ at all!

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**Homework**

*Web Problems:* “Section 4.3 - 3/4” is on the Wednesday when we return from break.