Math 120: Handout 02/16/11

Alex Kasman
Department of Mathematics/College of Charleston

Key Ideas from Section 3.9: Related Rates

- Remember that the derivative of a function describing something in the real world gives the *rate* at which that something is changing. In some situations, two different rates can be *related* to each other. (That means that you can't change one without changing the other.) For example, if you are blowing up a spherical balloon, the volume of the balloon and the radius of the balloon are each changing at some rate. They are not changin at the *same* rate, but these rates are necessarily related.
- The relationship between the two rates can be complicated. Often the relationship depends on the values of the changing quantities as well as on the rates. For instance: consider a square whose sides are increasing at a rate of 1 inch per minute. How quickly is the area changing? It depends on the length of the sides! When the square is larger the area increases more quickly.
- In this section (3.9) we learn how to find the value of one rate of change when we know the value of another related rate. (For example, when blowing up a balloon, we usually know how quickly the *volume* is changing – because we add gas to the balloon at a certain rate of volume/time – and could use this method to figure out how quickly the radius is increasing.)
  (All of the situations we will consider here will be cases in which all of the changing values are functions of the variable t, representing *time*. As we know from the last class, there is no reason that we could not consider rates with respect to some other independent variable. In theory, the techniques we learn today could be applied to those situations as well. I suppose it is just to keep things simple for you that we are making this restrictive assumption.)
- The method for finding the formula relating two rates is this:
  - Find an equation relating the two changing quantities (not their derivatives.)
  - Take the derivative of this equation (implicit differentiation) with respect to time (t). This will give a new equation involving all of the rates of change.
  - Plug in all known values of these quantities and their rates of change. (It is important that this step be done last and not before taking the derivative.)
- It is very often useful to draw a diagram before attempting to solve a related rates problem. Once you have the diagram, you will have to give a name to any changing quantities that have not already been assigned a name in the question.

Question 1: An tanker ship is leaking oil, which is expanding in a big circle around it. Since the oil is leaking out of the ship at a constant rate, the area of the circle is increasing by 10 square feet every second. How fast is the radius of the circle increasing when it is 200 feet? 300 feet?

Question 2: Two people start from the same point. One walks east at 3 mi/h and the other walks north at 2 mi/h. How fast is the distance between the people changing after 15 minutes?

- Note that solving these questions often requires you to make use of other pieces of information that you are expected to know from previous math classes (such as trigonometry), science (what is a shadow?) or even life outside of school (boats, lighthouses, ladders, etc.)

- ***Common mathematical sources for the equations:*** Pythagorean Theorem, area or volume formulas, *"SOHCAHTOA"*, equality of ratios of sides of similar triangles, etc.

Homework

Web Problems: The set "Section 3.9 - 2/16" is due on Friday, but I recommend that you start *immediately* and ask me about the hard ones in class tomorrow.